Iterative Kriging-based RBDO Methods for Expensive Black-Box Models

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Reliability-Based Design Optimization (RBDO) in electromagnetic field problems requires the calculation of probability of failure leading to huge computational cost in the case of expensive models. Three different types of RBDO approaches using kriging surrogate model are proposed to overcome this difficulty by introducing an approximation of the objective and of the constraints. These methods use different infill searching criteria to add new samples in the process of optimization or/and in the reliability analysis. The enrichment criteria and the best suited enrichment strategies are discussed in this communication. These approaches are compared in terms of number of evaluations and accuracy of the solution.

Index Terms**—Infill searching criteria, kriging model, reliability analysis, reliability-based design optimization.**

I. INTRODUCTION

RELIABILITY-BASED DESIGN OPTIMIZATION (RBDO) approaches can be divided into Double-Loop (DLM), Single-Loop (SLM) and Sequential Decoupled Methods (SDM). They have emerged in the past few decades and become more and more popular in electromagnetics owing to their ability of tackling the uncertainties. However, for expensive black-box models, the computational burden becomes unbearable.

To overcome this issue, iterative kriging surrogate models have been proposed to reduce the number of evaluations [1]. Infill Searching Criterion (ISC) was used with the aim of improving the quality of the surrogate model, and searching for the solution of the optimization problem.

With the purpose of enhancing the efficiency, different strategies including the choice of the ISC and its position for samples enrichment in the optimization process are investigated in this paper for each type of RBDO approaches. An analytical example and a transformer modelled with the finite element method are used to compare with normal RBDO without kriging models and highlight the most effective strategy.

II. INFILL SEARCHING CRITERIA

Generally, RBDO can be considered as a combination of deterministic constrained optimizations and reliability analysis. The criterion called Expected Improvement (EI) [2] is widely used for optimization without constraints. For constrained problems, an extended method consists in multiplying the value of EI by the probability that the point is feasible. However this probability of feasibility (PF) may prevent the sampling on the constraint boundary where the deterministic optimum may lie. Another constraint handling method is Expected Violation (EV) but the number of candidate points to evaluate can be very large. An alternative method is to use the predicted value of the constraint functions directly as constraints in the infill sub-problem where the objective is to maximize EI only. This could be more accurate than mixing PF or EV with EI in the objective function [3].

In addition, with the aim of searching the global deterministic optimum, locating the global optimum is more important than improving the accuracy of the kriging model. However as EI is highly multimodal, a local minimum is often found. To be sure to find the global solution, more attention should be paid on the infill criterion. The Weighted EI (WEI) criterion proposed in [4] seems to be more suitable as it adds weights into EI expression. Results show that small value of weight prevents WEI from converging to a local minimum.

However, in constrained optimization, WEI requires an initial sampling inside the security domain to start the improvement. To avoid this issue, a Modified WEI (MWEI) combined with the surrogate objective function is proposed:

$$MWEI = WEI - \omega \hat{f} \tag{1}$$

where ω is the same weight as in WEI and $\hat{f}$ is the surrogate objective function. For searching the global optimum, the weight is typically taken equal to 0.1.

III. INFILL STRATEGIES FOR RBDO METHODS

A. Double-loop method

DLM like Performance Measure Approach (PMA) [5] has a nested structure, the outer loop seeks for the optimum and the inner loop analyzes the reliability.

There are two places where ISC can be used to improve the accuracy of the kriging model: outer loop and inner loop. As outer loop is an optimization with inequality constraints, the criterion MWEI in (1) and the meta-models of constraints can be used. Additionally for the inner loop, EI is used directly with the original constraints because the equality constraints of inner loop are on variables. However as the two loops are nested, the enrichment in inner loop can bring out thousands of model evaluations. To check it, two strategies are proposed: The first one (PMA1) adds new samples only inside the outer loop, whereas the second (PMA2) enriches inside both the outer and inner loops.

B. Single-loop method

For SLM like Single Loop Approach (SLA) [6], the main point is that the model in the inner loop is replaced by an approximation based on a first order Taylor expansion. The probability of failure is then approximated to avoid the numerous evaluations required for reliability analysis. Also MWEI can be used straightly in this optimization. It's important to note that due to this approximation, the precision is low, so it's expected that with a surrogate model, two parts of the uncertainties will be superposed and the accuracy will be further reduced.

C. *Sequential decoupled method*

SDM like Sequential Optimization and Reliability Assessment (SORA) [7] is based on a serie of sequential deterministic optimizations and reliability assessments. The main point is to shift the boundaries of constraints to the feasible direction based on the reliability information obtained in the former iteration. The first deterministic optimization aims at searching the global optimum. Reliability assessments are conducted after to locate the Maximum Performance Target Points (MPTP) which corresponds to the desired probability of failure. Then new optimization is solved by taking into account the shift computed with MPTP.

Three strategies are proposed. The first one (SORA1) use MWEI and meta-models of constraints to add new points during each deterministic optimization. Then for reliability assessments, EI and the surrogate constraints for ISC sub-problems are applied to enrich near MPTP. This enrichment improves the accuracy of constraint boundaries in the vicinity of the current design point. It's more rational and less expensive than improving the accuracy for the whole domain.

The second strategy (SORA2) differs from the first one by the fact that no enrichment of the kriging models is made during the optimization at iterations higher than one. Indeed, it seems to be more important to add samples on the constraints boundaries and the solution of the optimization at all iterations except first is distant from those boundaries.

For the third strategy (SORA3), if the optimum found in k-th cycle is close to any of the other $k-1$ cycles, as the former reliability assessments have already added points in this region, the accuracy is considered to meet the requirement so there is no need to add new samples. The proximity criterion is:

$$\left\| d^k - d^i \right\| < \beta_t \sigma, \quad i = 1, \dots, k-1 \qquad (2)$$

where d^i is the deterministic optimum found by the i-th cycle, σ is the standard deviation of the input parameter, and β_t is the target reliability index. If the criterion is satisfied, the meta-model will be used straightly and only MPTP are evaluated. For the parts of deterministic optimization, it takes the same strategy as the second strategy.

IV. Applications

A. *Mathematical example*

To assess the efficiency of kriging-based RBDO methods, the analytical problem in [8] with two variables and three constraints is analyzed. The results are given in Table 1 with an initial sampling of 20 points. For comparison purpose, results given by RBDO methods with the original problem are also presented. All the iterative kriging-based RBDO methods lead to a reduced number of evaluations. SLA is not accurate enough because of the approximation used to simplify the reliability analysis. As mentioned, PMA with infill during inner loops requires thousands of samples to evaluate. The other strategy of PMA is faster but as it doesn't add samples in the vicinities of MPTP, it's not accurate enough. Kriging-based SORA strategies lead to the best result. The third strategy is the most efficient.

TABLE 1
Results of Mathematical Example Using Different Strategies

Strategy	Number of evaluations	Optimal solution	Optimal value
SLA(exact model)	165	[2.2512; 1.9677]	-1.9953
PMA/SORA (exact model)	3183/531	[2.2513; 1.9691]	-1.9945
SLA	26	[2.2466; 1.9617]	-1.9996
PMA1	29	[2.2494; 1.9649]	-1.9972
PMA2	1804	[2.2513; 1.9691]	-1.9945
SORA1/SORA2/SORA3	142/97/45	[2.2513; 1.9691]	-1.9945

B. *Finite element example*

A safety isolating transformer modelled with the finite element method presented in [9] is used for comparison. The optimization problem includes 7 variables and 8 constraints.

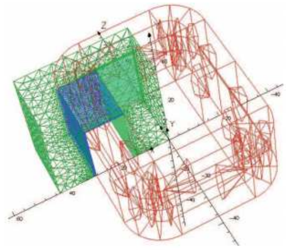

Fig. 1. The finite element model of the safety transformer.

Comparison of kriging-based RBDO methods and the conclusion will be presented at the conference.

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